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Logicism and Carnap's *Logical Syntax* *

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Summary
Although logicism played a significant role in Carnap's philosophical thinking, the relation of his philosophy of mathematics to the main tenets of the logicist tradition is complex and variable. This paper examines one aspect of this relation by discussing the following question: What elements of Carnap's *Logical Syntax of Language*, if any at all, indicate a real commitment to that tradition? It will be shown that although important aspects of Frege-Russell logicism are incorporated into the framework developed in that book, it is nonetheless impossible to define a position within that framework which deserves to be called "typically logicist".

1. Introduction

In his intellectual autobiography, Rudolf Carnap mentions the logicism of Frege and Russell as being one of the main influences on his philosophy of mathematics. In one chapter, in which he recalls the discussion on the foundations of mathematics in the Vienna Circle, he writes:

> I had learned from Frege that all mathematical concepts can be defined on the basis of the concepts of logic and that the theorems of mathematics can be deduced from the principles of logic. Thus the truths of mathematics are analytic in the general sense of truth based on logic alone. (Carnap 1963, 46)

Carnap explains the philosophical significance of this reductionist conception of mathematical truth for logical empiricism in that "it became possible for the first time to combine the basic tenet of empiricism with a satisfactory explanation of the nature of logic and mathematics." (Carnap 1963, 47) According to the autobiography, the idea was as follows: The reducibility of mathematics to logic makes it possible to extend

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Wittgenstein's account of the tautological nature of logical truth to the truths of mathematics. This shows that "all valid statements of mathematics are analytic in the specific sense that they hold in all possible cases and therefore do not have any factual content." (Carnap 1963, 47) Mathematical truths are therefore empty results of the representational function of language and an explanation of their necessary validity needs no principles of a kind inconsistent with an empiricist point of view.

Around 1930, Carnap published a series of papers in which he tried to defend this classical version of logicism. These papers are a straightforward attempt to vindicate empiricism by working out such a reductionist account of mathematical truth.[1] The relation of his philosophy to Frege-Russell logicism is however not as simple as his autobiography suggests.

On the one hand, Carnap's early philosophy cannot be placed squarely within the tradition of Frege-Russell logicism. Indeed, the thesis of the reducibility of mathematics to logic as well as logicism more generally played some role in Carnap's earliest writings: Already in one of his first philosophical writings, an unpublished essay written in 1920, he maintained that all of mathematics can be derived from deductive logic (cf. Carus 2007, 97), and in his first book *Der Raum*, published in 1922, he subscribed to this basic tenet of logicism as well (Carnap 1922, 62).[2] Nonetheless, as is well known, his early publications, from *Der Raum* to *Der logische Aufbau der Welt*, were strongly influenced by neo-Kantianism and by Husserl's phenomenology. In 1920 Carnap even adopted Natorp's criticism of Russell's logicism to the effect that deductive logic cannot be the primary sector of logic since it presupposes the notion of an object (cf. Carus 2007, 97). Carnap rejected this criticism soon afterwards. But also after he had given up the idea that deductive logic is to be grounded somehow in an antecedent transcendental explanation of the notion of an object, some years passed before he made the main tenets of Frege-Russell logicism a real topic of his philosophical investigations. Even in his first major work, the *Aufbau*, written largely in the years 1922–1925, he neither discusses in detail nor attaches great importance to the reduction

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[1] See Carnap (1930a), (1930b), and (1931). By calling the position "classical", I do not intend to suggest that it is the position originally held by Frege or Russell. A logicism of the type sketched above depends on Wittgenstein's analysis of the nature of logic. Such a position is therefore the result of an important break with the Frege-Russell tradition; compare Uebel (2005, 179).

[2] For a detailed discussion of the influence of logicism on Carnap's earliest writings, see Carus (2007, 97–105).

of mathematics to deductive logic. In effect, he simply affirms the possibility of this reduction (Carnap 1998, § 107).

On the other hand, in his book *Logical Syntax of Language*, published in 1934,[3] Carnap seems to reject essential principles not only of his logicism in 1930 but also of Frege-Russell logicism as well.[4] Due to its commitment to a purely syntactic conception of logic, the standpoint of that book may, on superficial consideration, even lead to the suspicion that Carnap has rejected logicism in favour of Hilbertian formalism.

The relation of Carnap's philosophy to the logicist tradition is thus by no means trivial. This paper discusses one aspect of this relation by trying to answer the following questions: What elements of *Logical Syntax*, if any at all, indicate a real commitment to the logicist tradition? What should we make of Carnap's attempt in *Logical Syntax* to defend a requirement which he labels "logicist" and which he regards as a precise version of the main content of the traditional logicist positions (325–327)[5]? Does Carnap's avowal of logicism in 1934 indicate any real continuity in content or is it merely an expression of his high esteem for those men who led him on the way to his mature philosophy of mathematics?

2. A place for logicism?

First, I will sketch Carnap's philosophical development between 1931 and 1934,[6] after which I will outline, in a preliminary way, why one may well wonder whether there is still a place for logicism in *Logical Syntax*.

In January 1931, Carnap began treating the logical language as a system of uninterpreted marks. He defined the central notions of logic in purely syntactical terms, that is, in terms referring exclusively to the type and the order of signs. The concept of meaning was rejected as obscure. Later, in the autumn of 1932, he adopted his principle of tolerance, which is stated in *Logical Syntax* as follows: "*In logic, there are no morals*. Everyone is at liberty to build up his own logic, i.e. his own form of language,

[3] The German original *Logische Syntax der Sprache* was published in 1934; an English translation first appeared in 1937.
[4] It is therefore no surprise that some exponents of the logicist tradition regarded the conception advanced in *Logical Syntax* as completely mistaken. A nice illustration is provided by Russell's comments on that book in the introduction to the second edition of *Principles of Mathematics*; compare Russell (1937, xii).
[5] Unless stated otherwise, indications of page numbers always refer to Carnap (1937).
[6] The following sketch of Carnap's intellectual development between 1931 and 1934 relies heavily on Awodey and Carus (2007).

as he wishes." (52) With this principle Carnap responded to his insight that there is no such thing as an absolute notion of logical truth for a given language (cf. Awodey and Carus 2007, 38). It amounts to the proposal that the philosopher should give up the idea that his acceptance of a language form can be guided by criteria other than by its usefulness for given purposes.[7]

Thus, in the course of this intellectual development, Carnap not only rejected the concept of meaning, he also dismissed attempts to justify claims about the real nature of mathematical truth. A tolerant philosopher does not try to determine what mathematical truth really is. He only develops languages which may be regarded as capturing, to a certain degree, the ideas expressed by traditional philosophical claims, and then judges the practical utility of these languages. Classical logicism, however, is nothing else but an attempt to establish the absolute validity of classical mathematics on the basis of the meaning of the mathematical signs: The purely logical definitions of these signs are intended to clarify their meaning, and the purely logical proof of the truths resulting from the elimination of the defined signs are intended to show that the truth of a mathematical theorem is already guaranteed by the meaning of its signs.

Logical Syntax is mainly an attempt to develop a conceptual framework which enables us to discuss in a precise way the logical properties of various languages as well as the relations holding between these properties. This framework is shaped by the principle of tolerance and a syntactic conception of logic and, as just shown, therefore excludes a particular classical understanding of the logicist reduction. In addition, the two sample languages construed in *Logical Syntax*, Languages I and II, contain primitive signs and primitive principles classified by Carnap as genuinely mathematical in 1930. In fact, in 1934 he shows no interest at all in the question of whether a mathematical system is constructed on a basis he would have regarded as having a purely logical character in 1930:

[7] In *Logical Syntax,* languages or calculi are understood as systems of formation and transformation rules. Formation rules determine the conditions under which a string of signs is a sentence, while transformation rules determine under what conditions the transformation of one or more sentences into others may be allowed (4). According to Carnap, both kinds of rules are to be stated in purely syntactical terms, i.e. without reference to the meaning of expressions, but solely with reference to the kinds and order of symbols from which the expressions are constituted (2).

> Whether … only logical symbols in the narrower sense are to be included amongst the primitive symbols (as by both Frege and Russell) or also mathematical symbols (as by Hilbert), and whether only logical primitive sentences in the narrower sense are to be taken as L-primitive sentences,[8] or also mathematical sentences, is not a question of philosophical significance, but only one of technical expedience. In the construction of Languages I and II we have followed Hilbert and selected the second method. (327)

Prima facie, it seems therefore rather implausible to suggest that logicism or the reducibility of mathematics to logic still plays an important role in the project of *Logical Syntax*. However, as already mentioned, Carnap's *Logical Syntax* offers a characterisation of the logicist position. According to that book, logicism is the demand that a language used in science contain application rules for mathematical signs in descriptive sentences (327). Thus, the position is equated with the need for an encompassing framework containing the empirical and the formal sciences in a way which allows the application of mathematics to empirical matters as well. But can we really regard such an integrality requirement as typically logicist? Does this requirement represent the core of Carnap's commitment to the logicist tradition in 1934?

In order to answer these questions, it is necessary to keep an important distinction in mind. If we suppose, as I will do in this paper, that Carnap's theory in *Logical Syntax* about the nature of scientific philosophy is generally correct, a philosopher can only pursue one of the following two projects: On the one hand, he can attend to a discipline which Carnap calls "general syntax". It is the aim of this discipline to define a system of logical concepts in such a comprehensive way that these concepts may be related to any language whatsoever (167). General syntax is thus an attempt to develop a conceptual framework for the discussion of the logical properties of all possible languages. On the other hand, a tolerant philosopher can set himself the task of constructing languages within the framework of general syntax, investigating their logical properties, and evaluating their usefulness for given purposes. He may carry out his construction with the intention of developing a language to be used in science or with the intention of

[8] The L-rules of a calculus are its logico-mathematical transformation rules. Besides such L-rules, a calculus may also contain physical transformation rules or P-rules. For Carnap's attempt at distinguishing syntactically between these two types of rules, see (180–181).

clarifying the content of a philosophical position. With his Language I, for example, Carnap develops a system in which finitist or constructivist tendencies in the philosophy of mathematics find their realisation.[9] With his Language II, he tries to construct a system adapted to the needs of mathematical physics.

In examining the relation of *Logical Syntax* to classical logicism, we may therefore either ask whether a philosopher pursuing the first of these two projects can be seen as working in the logicist tradition or whether this may be true concerning the second. This means that we must address the following two questions separately: i) Are there aspects of Carnap's views concerning the construction of the general syntax framework which indicate a genuine commitment to the logicist tradition? ii) Is it possible to formulate useful languages within that framework which capture important aspects of classical logicism? In answering question i), we must discuss questions such as the following: Are Carnap's views about how precisely defined languages are to be constructed shaped by convictions characteristic of the logicist tradition? Are his definitions in general syntax guided by such convictions? In order to answer question ii), however, we must ask questions like the following: Is it possible to develop a language within the framework of *Logical Syntax* which may be regarded as a logicist language? If such languages are possible, are there practical reasons to prefer these languages under certain circumstances? The following chapters will mainly focus on question ii), although question i) will be discussed along the way.

3. How to analyse classical logicism

In order to answer the second of the two questions mentioned above, we must take a thesis championed by proponents of a logicist position as our starting point. Then we must try to accommodate the chosen thesis in some way within the framework of *Logical Syntax*. But how should we proceed? In this chapter I will provide a general discussion of the method to be used. In chapters 4–7 I will then apply this method to claims maintained in the logicist tradition. As an illustration, in this chapter I will use the following claim, defended by, for example, Russell:

[9] In Carnap's opinion, it is impossible to decide, however, whether these tendencies are precisely realised in I. This is due to the fact that the usual formulations of these tendencies are too vague to determine the construction of a language in detail (46).

1) Numbers are classes of classes of things.[10]

Since claim 1), as is common for most claims maintained by philosophers, is not precisely formulated according to Carnap's standards in 1934, it must first of all be clarified, and as a first step in rendering 1) precise, we must decide what kind of sentence it is. Since 1) is not a pseudo-sentence, that is, a sentence having no logical content, there remain according to *Logical Syntax* only the following three possibilities: 1) is either an object-sentence, a syntactical sentence, or a pseudo-object-sentence. Object-sentences refer to objects occurring in the special sciences. Syntactical sentences refer solely to the kinds and the order of symbols. Pseudo-object-sentences, finally, "are formulated as though they refer (either partially or exclusively) to objects, while in reality they refer to syntactical forms, and, specifically, to the forms of the designations of those objects with which they appear to deal." (285) A pseudo-object-sentence is therefore always translatable into a syntactical sentence speaking about the designations of the objects the original sentence seemed to deal with.[11]

Since 1) belongs, according to Carnap, to the last of these three types, 1) is to be clarified by translating it into the language of syntax. But how are we to translate these pseudo-object-sentences? If someone uses such a sentence in a discussion, we can ask for a syntactical sentence he regards as expressing his opinion (302). Thus, if someone champions 1) but rejects all syntactical sentences as adequately expressing his opinion, he disqualifies himself as interlocutor. However, if we are examining a pseudo-object-sentence found, for example, in a treatise on the foundations of mathematics, we must check to see whether the sentence contains any indications as to how it may be translated into the language of syntax. If this is not the case, the sentence falls outside the realm of science and is beyond discussion. If it is the case, we must find a translation by considering these indications as well as the ordinary use of language and the definitions given by the author. However, in general there will not be a uniquely correct translation. Since philosophical theses cannot, for the most part, be understood univocally, we will generally have to choose between various translations into a syntax-

[10] This example is also discussed in *Logical Syntax*, compare (300).

[11] It is quite clear that these descriptions of the three types can lay no claim to precision (277). They are even expressed in pseudo-object-sentences. However, these vague formulations suffice for my undertaking.

language which differ not only in content from each other but also from the original sentence as well (302).

Let us now suppose that we have decided to accept the following syntactical sentence as a translation of 1):

2) Numerical expressions are class-expressions of the second level.

This translation however admits still different interpretations: On the one hand, we can interpret 2) as a proposal to construe a calculus, that is, a proposal to the effect that for certain purposes we should use a language in which numerical expressions are class-expressions of the second level. If 2) is understood in this way, a discussion about its truth is quite mistaken; it is then only possible to investigate the utility or the consequences of the proposal. On the other hand, we can interpret 2) as an assertion about syntactic properties. In that case, we must specify the language, or the kind of languages, the assertion is intended to hold for (299–300). We can interpret 2), for example, as the wrong assertion that 2) holds for every language, but also as the correct one that there are languages for which 2) holds. Since most of the sentences relating to the so-called foundational problems are, according to Carnap, of the same type as 1) (322–323), they are thus, for the most part, to be understood as proposals to construe calculi or as assertions about syntactical properties of languages.

Given these distinctions, I am now in a position to state Carnap's theory of clarification in *Logical Syntax*: An object-sentence is to be clarified by a logical analysis of the language of the special science to which that sentence belongs. A pseudo-object-sentence is to be clarified by translating it into a syntactical sentence. The resulting sentence is then to be further clarified in the same way as other syntactical sentences, that is, by specifying whether we interpret it as a proposal or as an assertion, and by making the necessary completions.

Thus, one possible result of clarifying 1) runs as follows:

3) It is possible to construe calculi in which numerical expressions are class-expressions of the second level.

Sentence 3) is a correct and precise version of a logicist thesis. But can 3) be used, together with other claims, to define logicism within the framework of *Logical Syntax?* In my opinion, this is not the case: Carnap distinguishes between questions of philosophical significance and questions of technical expedience (327), and since it is our goal in characterising logicism to define a philosophically significant position, we should not use claims concerning only questions of technical expedience. In addition, according to *Logical Syntax,* it is not even a question of philosophical significance whether only logical signs in the narrower sense or also mathematical signs are taken as primitive. Therefore, it is clear that the same holds for the question of whether to use a calculus for which 2) holds, or a calculus in which the numerical expressions are of a different level.

However, how can we distinguish precisely between philosophical significance and technical expedience? Clearly, if the difference between two language forms is of philosophical significance, then it should make a difference, with regard to some practical purposes, which of the two forms is used. However, the preference for language form L as opposed to form L' in pursuing certain purposes may still be due to two different reasons: i) Only the adoption of L, and not also of L', is appropriate to the purposes at hand. ii) The adoption of both forms is appropriate to these purposes, but L is simpler and more convenient than L'. Thus, the difference between two calculi is of philosophical significance if there are purposes such that, with regard to them, the adoption of one of the two calculi is preferable to the adaption of the other for reason i). Otherwise, the difference concerns at most technical expedience, and it does so if one of the two calculi is nonetheless preferable to the other for reason ii).

But let us now consider a claim to the effect that languages with certain syntactical properties are possible. Under what conditions should we regard such a claim as expressing a philosophically significant position? Clearly, this is only the case if the following two conditions are satisfied: First, we can think of purposes such that, with regard to them, it is reasonable to adopt a language having the syntactical properties in question. Second, not all languages possess these properties. The reason for adding the second condition is that properties possessed by all languages cannot be used to make a distinction within the framework of general syntax. Claims that concern such properties do not distinguish a position within that framework but characterise the framework itself.

In the rest of this paper, I will apply the method discussed in this chapter to claims defended in the logicist tradition and, especially, to those claims regarded by Carnap as characteristic of logicism in 1930. Thereby I will try to establish the following thesis: If we clarify that position within a framework shaped by the principle of tolerance and a syntactic conception of logic, and if we ignore all purely technical questions, only one requirement remains – it is the demand for a comprehensive language containing the empirical and the formal sciences in a way which allows the application of logic and mathematics to empirical matters as well.

4. Mathematics as a part of logic

Carnap's defence of logicism in 1930 was mainly an attempt to establish the following claim:

1) All of mathematics is reducible to logic and because of this reducibility tautological.[12]

In characterising logicism by 1), we picture the logicist as trying to show that mathematical truth is logical truth and for this reason tautological. Thus, in order to reach his aim, the logicist must on the one hand ensure that the basis of his reduction consists of tautologies. On the other hand, he must construct the system of mathematics in such a way that all mathematical truths result from this basis by means of a series of steps which generate tautologies from tautologies. But what does it mean for a sentence to be tautological? Carnap defined a tautology in 1930 as a sentence true for any possible assignment of truth-values to its component sentences.[13] It is a consequence of this definition that a tautology rules out no possible case, and from this Carnap inferred that such a sentence holds in virtue of its form alone (Carnap 1930b, 305–306).

To all appearances at least, there is therefore an important difference between 1) and the following claim:

2) All of mathematics, like pure logic, is tautological.

Of course, 2) is implied by 1). But 1) also contains the idea that the tautological nature of mathematics is a consequence of its reducibility to logic.[14] However, as will be

[12] Around 1930, Carnap characterised logicism using the following two claims: i) All mathematical concepts are reducible to logical concepts by means of explicit definitions. ii) All mathematical theorems are logically deducible from the basic principles of logic. Compare Carnap (1930a, 20) and (1931, 91–92). He then argued that it is a consequence of these claims that all of mathematics is tautological. Compare Carnap (1930a, 23) and (1930b, 306).

[13] Such a definition of the notion of tautology only seems adequate for propositional logic. However, Carnap does not discuss in his published writings from 1930 and 1931 how this definition can be extended to predicate logic.

[14] Some members of the Vienna Circle also tried to defend the idea that mathematics is tautological without asserting the reducibility of mathematics to logic. Schlick, for example, construed the notion of

shown in this chapter, this difference between 1) and 2) cannot be formulated in a philosophically significant way within the framework of *Logical Syntax*. The idea of a privileged logical basis, essential for Carnap in 1930, no longer has any content in 1934.

In order to establish this point, we have to clarify the claims 1) and 2). And as a first step in rendering them precise, we must decide what logico-mathematical system we assume 1) and 2) deal with: The sentences 1) and 2) are formulated as if there is only a single system of logic. However, if someone adopts the attitude recommended in the principle of tolerance, he will no longer maintain that there is the "true logic" (xiv). Instead, he will maintain that there is a multiplicity of possible logical and mathematical systems, no one more correct than the others.

Since the logicists typically advocate classical logic and mathematics, the most plausible approach is to interpret 1) and 2) as concerning these systems. However, we can also formulate part of the content of these claims in a general way. The logicist maintains not only that all mathematical truths are tautological, but also that all mathematical sentences are either true or false, and that all mathematical falsehoods are contradictions. Now, the resulting claim that all sentences of pure logic and mathematics are either tautological or contradictory can be formulated within the framework of *Logical Syntax* as a theorem holding for all possible languages. In 1934 Carnap uses the expression "analytic" in place of the expression "tautological" and tries to define the notions "analytic" and "contradictory" in such a way that an exact understanding is reached of what is meant by calling a sentence true (respectively false) on purely logical grounds. He defines "analytic" and "contradictory" along the following lines: A sentence of a language is analytic in that language if it results in accordance with the logico-mathematical transformation rules of the language from the sentential null class. A sentence of a language is contradictory in that language if every sentence of the language results from it in accordance with the logico-mathematical transformation rules (182).

When combined with Carnap's general characterisation of the logico-mathematical sentences of a language, these definitions lead to the following result:

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tautology as meaning a sentence is true, independent of experimental facts. Therefore, concerning a proof of the tautological nature of mathematics, he regarded it as irrelevant whether mathematics is a part of logic or not (cf. Goldfarb 1996, 219–223).

3) The logico-mathematical sentences of any language are analytic or contradictory in that language (184).

The fact that Carnap accepts this constraint of the logical determinateness of all logical and mathematical sentences indicates a real commitment to the logicist tradition. Not only is that constraint motivated by the idea that logic and mathematics are independent of all empirical facts (cf. Awodey 2007, 245), it is also a formal version of the idea that logical and mathematical truths (respectively falsehoods) are of the same kind. An important aspect of classical logicism is therefore built into the framework of *Logical Syntax* itself. In fact, Carnap even regards the correctness of 3) as an adequacy condition for any acceptable definition of "analytic" and "contradictory" in general syntax (179). But clearly, this must also mean that we cannot use 3) to characterise a philosophically significant position within that framework.[15]

Let us therefore turn to the suggestion that the claims 1) and 2) are to be interpreted as dealing with the systems typically defended by the logicist, that is, classical logic and classical mathematics. Clearly, this is the most plausible approach to defining a philosophically significant position with the help of 1) and 2). However, Carnap does not draw a formal distinction between logical symbols in the narrower sense and mathematical symbols, that is, between those signs we would usually regard as belonging to pure logic and those considered genuinely mathematical signs (327). With the help of the concepts actually defined in *Logical Syntax,* it is therefore impossible to express directly the idea that classical mathematics can be constructed on a purely logical basis. We can only specify a basis we would usually regard as purely logical by listing some primitive signs and rules of transformation. The most promising translations of 1) and 2) into the language of syntax run therefore roughly as follows:

[15] For this reason the following characterisation of logicism offered by Ricketts is highly misleading: "I maintain that … logicism also has the status of a proposal: it is the recommendation that candidate languages of science be restricted to those in which sentences constructed from just the logico-mathematical vocabulary are calculus-determinate." (Ricketts 2007, 217) Such a recommendation is clearly empty as long as we accept Carnap's adequacy condition for an acceptable definition of analyticity in general syntax. Therefore it can only concern the question of how to define that notion in general syntax. However, Ricketts regards that recommendation as Carnap's explication of logicism in *Logical Syntax* (Ricketts 2007, 218), and this is wrong: If we try to explicate that position, we have to formulate a proposal to the effect that we should choose languages having certain syntactical properties from the domain of those languages which are construed according to the constraints of *Logical Syntax*.

4) On the basis of a system called a system of classical logic, it is possible to construct a calculus containing as analytic sentences all those sentences which are regarded as correct in classical mathematics.[16]

5) It is possible to construct a calculus which contains all those sentences regarded as correct in classical logic and mathematics as analytic sentences.[17]

Now, whereas there seemed to be an important difference between 1) and 2), there is no such difference between the formal versions 4) and 5) of these claims. This point can be established as follows: Let us call the calculi constructible according to 4) CL-calculi, and let us suppose that there is such a difference between 4) and 5). Obviously, the CL-calculi are only distinguished thereby from the calculi constructible according to 5) in that the calculi of the second type may rest on a basis containing primitive mathematical elements as well. It must therefore be a question of philosophical significance whether to use a basis containing only logical elements in the narrower sense or a basis containing genuinely mathematical elements as well. However, according to Carnap, this is simply not the case (327).

Therefore, the additional content which 1) seems to contain in contrast to 2) is of no philosophical significance, i.e. whether we define a position on the basis of 4) or of 5) makes no difference. But can we regard such a position as philosophically significant? I will defer this question to the next chapters. First, I will discuss the central premise of the argument developed just now.[18]

This argument rests on the claim that the choice between a CL-calculus and a corresponding calculus with a logico-mathematical basis has no philosophical significance. But do we have to accept that premise? The goal of the following reflections is to show that this question must be answered in the affirmative. However, since Carnap does not discuss the point in *Logical Syntax*, they are merely a sketch of an answer he might have given.

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[16] I will assume that 4) holds. The question of whether this assumption is justified cannot be examined in this paper. However, it is clear that, in addition to such rules as are usually stated in systems of higher order predicate logic, the basis of such a calculus must contain indefinite transformation rules as well. For Carnap's discussion of this point, see for example (98–101).

[17] It should be noted that formulations 4) and 5) are not completely acceptable according to the standards of *Logical Syntax*. They contain the imprecise expression "regarded as correct in classical logic and mathematics".

[18] Richardson (1994, 76) stresses that logicism is rejected in *Logical Syntax* as an attempt to define mathematical vocabulary in terms of an antecedently understood logical vocabulary. However, he justifies his claim only by the fact that Carnap does not distinguish between logical symbols in the narrower sense and mathematical symbols.

In equating logicism with 1), we picture the logicist as trying to reduce mathematics to logic because, in his eyes, the non-obvious, yet interesting thing to know is whether mathematics, like pure logic, is tautological. But why should we, from a philosophical viewpoint, prefer one of two reductionist projects, both of which concern the question of whether all of mathematics possesses a property P? Clearly, our preference must be based on the fact that only the basis of the preferred project has an "indicating" property for P, that is, a property P', such that anything reducible to a basis having P', has P. Otherwise, we can base our preference merely on reasons of technical expedience.

If we characterise logicism by 1), a proponent of that position regards logicality in the narrower sense as an indicating property for tautologicity. However, in *Logical Syntax* Carnap clarifies the concept of tautology by his definition of analyticity and, according to that definition, logicality in the narrower sense is not the only indicating property for analyticity. The transformation rules of Carnap's Language II are, for example, clearly not logical in the narrower sense. Nonetheless, the consequences of the sentential null class are analytic in II. This is due to the fact that these rules are logical in a wider sense, which includes mathematics as well. Moreover, since all theorems of classical mathematics result in II from the null class, classical mathematics is analytic in II due to its reducibility to a logico-mathematical basis. From a philosophical viewpoint, there is therefore no reason to prefer a CL-calculus to the project of construing classical mathematics within such a language as II.[19]

In addition, we can also apply the criterion for the distinction between philosophical significance and technical expedience, as developed in chapter 3. If that distinction must be drawn along these lines, the choice between a CL-calculus and such a language as II will clearly concern only a question of technical expedience. At least I can think of no practical purposes for which it would only be appropriate to use a CL-calculus and not also a corresponding language containing primitive mathematical elements as well.

Thus, nothing is left in *Logical Syntax* of the idea that the tautological nature of the mathematical truths is a consequence of their reducibility to a basis which is purely logical in some distinguished sense. The concept of analyticity is simply not, as the

[19] Since we must therefore choose between CL-calculi and such languages as II on reasons of technical expedience alone, it seems quite clear that we should prefer languages of the second type: They are simpler than CL-calculi. Carnap's procedure in *Logical Syntax* seems therefore completely reasonable.

concept of a tautology seemingly was, bound up with the idea of a reduction to such a privileged basis.

5. The analyticity of mathematics

As was shown in the last chapter, the core idea of classical logicism – that mathematics is tautological because of its reducibility to pure logic – simply amounts to the following claim within the framework of *Logical Syntax*:

1) It is possible to construct a calculus which contains classical logic and mathematics in such a way that all sentences regarded as correct in these disciplines are analytic.

However, 1) is redundant in one respect. In fact, there is no significant difference between 1) and the following thesis:

2) It is possible to construct a calculus which contains classical logic and mathematics.

The point can be established by considering the following question: What does it mean for a calculus to contain classical logic and mathematics? Although Carnap offers no explicit answer in *Logical Syntax*, it is nonetheless clear that in his opinion such a calculus must contain those sentences regarded as correct in classical logic and mathematics as analytic sentences. In particular, such a calculus must contain classical logic and mathematics as logico-mathematical systems. However, this demand is not as trivial as one might suppose. Gödel, it might be argued, has shown that some correct mathematical sentences cannot be established by the method of deduction of classical mathematics. Since a calculus formalizing classical mathematics would then contain correct sentences which are not analytic, it would not be a logico-mathematical system according to Carnap's standards. However, in Carnap's opinion, Gödel's results do not show that the method of deduction of classical mathematics is incomplete. These results merely establish the incompleteness of certain attempts to formalize that method.[20]

In fact, Carnap tries to define "analytic" and "contradictory" for his Language II in such a way that the following condition is satisfied: A logico-mathematical sentence of II is analytic (respectively contradictory) if and only if it is regarded as correct

[20] For this reason, two methods of deduction are distinguished in *Logical Syntax*: the method of derivation, which is based on definite rules, and the method of consequence, which admits indefinite steps and infinite classes of premises (99–100).

(respectively false) in classical mathematics for purely logico-mathematical reasons (100–101, 124). In addition, it follows from Carnap's definitions that all logico-mathematical sentences of II are either analytic or contradictory (116). We have therefore to conclude that, in Carnap's opinion, all correct sentences of classical mathematics are correct on purely logico-mathematical grounds. Now, if we want to represent a language used in practice formally through a calculus, we must define the notion of analyticity for that calculus in such a way that exactly those sentences of the calculus are analytic which are correct on purely logico-mathematical grounds according to the customary understanding of the language (181). If a calculus is to be a formalization of classical logic and mathematics, we must therefore define its method of deduction in such a way that all those of its sentences which correspond to correct sentences of classical mathematics result from the sentential null class.

This does not mean that there is a uniquely correct definition of analyticity for such a language as II or a canonical meta-language to be used in defining this notion. Around 1932, Carnap realized that the notion is always relative to a more or less arbitrarily chosen meta-language (cf. Awodey and Carus 2007, 38). However, if we want to construct a calculus containing classical logic and mathematics, we must nonetheless use a suitably strong meta-language in which the concept of analyticity is defined in such a way that all the relevant sentences are analytic.

Let me summarize the conclusions reached so fas. The idea that logic and mathematics are tautological may be formulated within the framework of *Logical Syntax* as a proposal along the following lines: It is advisable with regard to certain purposes to use a language in which classic logic and mathematics are analytic. In addition, this proposal is philosophically significant since the use of such a language seems advisable in parts of science. However, one must not suppose that something of philosophical significance is gained by using the word "analytic" in this context. The argument adduced in this chapter shows that such a proposal may be expressed equally well by simply saying that it is advisable for such-and-such purposes to use a language containing classical logic and mathematics. And since the use of the word "analytic" suggests something else in an attempt to characterise a logicist position, we should obviously prefer this second formulation.[21]

Clearly, the fact that Carnap does not distinguish between these two phrasings of the proposal, as well as the fact that he regards all logico-mathematical sentences of a language as analytic or contradictory, indicates a real commitment to the logicist tradition. However, these points of contact with that tradition guide the construction of the framework of general syntax itself. And, as shown in this and the previous chapter, this means we must equate the core idea of classical logicism with a proposal to the effect that we should use a calculus containing classical logic and mathematics in pursuing certain ends.

6. The demand for a meaning determination

We can strengthen this proposal somewhat, however, by taking into account another aspect. In fact, Carnap maintained in 1930 that logicism is especially concerned with satisfying the following demand:

1) Mathematics is to be constructed in such a way that it is applicable in empirical science as well.[22]

But how can we formulate 1) syntactically? 1) is the demand for a calculus which allows and systematizes calculations with numbers of empirical objects and with measures of empirical magnitudes. Since rules which allow and systematize such calculations cannot be anything but certain formation and transformation rules, we can equate 1) with the following proposal:

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[21] For this reason, Friedman's attempt to formulate logicism within the framework of *Logical Syntax* is at least misleading. He writes: "Logicism … is the proposal to use both classical logic and mathematics in a formulation that makes it clear that logical and mathematical rules are of the same kind – that they are, in an appropriate sense, analytic." (Friedman 2004, 111) This characterisation of logicism seems to imply that, within the framework of *Logical Syntax,* we may also propose using classical logic and mathematics in a formulation leading to the suspicion that classical logic and mathematics are not analytic. However, as shown in this chapter, the only thing we may wonder about is whether a language contains these systems or not.

[22] More precisely, Carnap assessed this demand around 1930 as follows: In *Die Mathematik als Zweig der Logik* he not only claimed that logicism pays special attention to demand 1), but also considered 1) to be one of the fundamental ideas of Frege (Carnap 1930b, 309). In the discussion on the foundations of mathematics in Königsberg, Carnap described demand 1) as the demand of the physicist and he contrasted it both with the demand of the logician (raised by Frege, Russell, and Brouwer) and of the mathematician (raised by Hilbert). He pictured the logician as demanding that all signs of the language have a definite, determinable meaning, the mathematician as demanding the right to operate freely in an axiomatic manner (Hahn et al. 1931, 141). Carnap added that the systems of Frege and Russell satisfy this demand of the physicist, whereas this is not clearly the case for Hilbert's system (Hahn et al. 1931, 142).

2) For certain purposes it is advisable to use a calculus containing formation rules for the occurrence of mathematical symbols in synthetic sentences, together with transformation rules for such sentences.

Starting from claims Carnap regarded in 1930 as typically maintained by logicists, a proposal can therefore be formulated within the framework of *Logical Syntax* as follows:

3) For certain purposes it is advisable to use a total language containing not only classical logic and mathematics, but also syntactical rules for the use of logico-mathematical signs in synthetic sentences.

A position which makes proposal 3) is obviously philosophically significant in accordance with the criterion developed earlier. In addition, 2) is not only a formal version of the demand that applications of mathematics must be possible. In Carnap's opinion it is also a formal version of the following claim, which he regarded in 1930 as one of the central tenets of Frege-Russell logicism (Hahn et al. 1931, 141):

4) Mathematics is to be built up in such a way that all its signs receive a definite, determinable meaning.

In *Logical Syntax*, Carnap establishes that 4) is, in effect, the demand for syntactical rules regulating the use of mathematical signs in synthetic sentences with the following argument (§ 84):

The logicist demands that the meaning of the mathematical signs be determined, because he requires mathematics to be applicable to reality as well. He thus makes the following claim: The meaning of the mathematical signs must be determined in order that mathematics may be applied to reality.[23] This sentence is a pseudo-object-sentence, however, and is therefore to be clarified by translating it into a syntactic sentence. The result runs as follows:

5) By stipulating application rules for the mathematical signs, an interpretation of mathematics is effected (327).

Since application rules are formation and transformation rules for synthetic sentences containing mathematical signs, 5) can be established in general syntax: Carnap defines an interpretation syntactically as a translation of a language into another language (228),

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[23] One logicist who explicitly makes this claim is Frege. He writes in *Grundgesetze*: "How is it possible to make applications of arithmetical equations? It is only because they express thoughts." (Frege 1903, 100, my translation)

and application rules provide a translation of the mathematical language into a language comprising synthetic sentences as well. Now, since these rules do not belong to a calculus of pure mathematics but to the syntax of a total language, the logicist requires in effect that:

6) One must construct the syntax of a total language which contains both logico-mathematical sentences and the synthetic sentences of applied mathematics (327).

This argument, which supports the claim that the demand for a meaning determination amounts to 6), rests on the transition from a pseudo-object-sentence to a syntactical sentence. With this transition the relation between the applicability of mathematics and a meaning determination is clarified: If it is claimed that the meaning of the mathematical signs must be determined in order for mathematics to be applicable to reality, a determination of the meaning of these signs seems to be an indispensable pre-condition for any application. However, 5) shows that those stipulations regulating the application of mathematics also determine the meaning of its signs.

In 1934 Carnap uses requirement 6) to define the logicist position (327), and the theses discussed so far do not lead to a much more substantial account of classical logicist concerns within the framework of *Logical Syntax*. In fact, the only difference between Carnap's characterisation and the one discussed at the beginning of this chapter is that Carnap does not specify the mathematical system which is at stake. Nonetheless, if we presuppose the theory of clarification developed in *Logical Syntax*, these characterisations can be regarded as formal versions of some of the claims maintained in the logicist tradition. Of course, this does not yet mean that the continuities between a classical position and Carnap's version of it in 1934 suffice to justify his use of the label "logicism". Perhaps he should have chosen a different name. However, I will defer the discussion of this question to the final chapter of this paper. First, I will examine some additional theses which might be considered typically logicist.

It is important to note that a wide range of languages satisfies Carnap's syntactical version of the demand for meaning determination. This demand is, for example, satisfied by such logicist systems as that of *Principia Mathematica*: The definitions of these systems may be understood as providing an interpretation of mathematics in a total language which allows the formulation of all of empirical science (326). Thus, the system of *Principia Mathematica* still vindicates logicism in 1934, not because it establishes the reducibility of mathematics to logic, but because it provides application rules. However, Carnap's syntactical version of logicism is not only satisfied by these logicist language systems; Language II, for example, meets that demand as well.

We may therefore well ask whether it is possible to strengthen this demand for a meaning determination in some way. However, according to *Logical Syntax,* that demand simply asks for an interpretation. Since an interpretation is to be defined syntactically as a certain correlation between two languages (222–228), the most trivial rendering of that demand amounts to the claim that it is necessary to set up a certain correlation between the mathematical language and another language. Carnap strengthens this trivial rendering by incorporating the motives which lead logicists to require an interpretation, and he considers them to be requesting that the mathematical language be integrated into a total language of science. The only plausible way to further strengthen this demand is to include the idea that classical logicism tries to provide such an integration in a peculiar way, i.e. that position tries to give an interpretation of mathematics by defining the mathematical signs in such a way that they may be used directly in descriptive sentences. However, this seemingly important aspect of the original logicist concerns is, according to *Logical Syntax,* simply devoid of philosophical significance. In 1934 Carnap considers it a purely technical question whether to use a language containing no undefined number signs or such a language as II, in which the numerical expressions are introduced as primitive and applicable in descriptive sentences only by means of transformation rules (cf. Oberdan 1993, 163–164).[24]

7. The ideal of a universal language

Let us finally take a closer look at the thesis that there is a single universal language system which provides the framework, so to speak, for all discourse. This thesis may be

[24] This point can be clarified by considering the theory of implicit definitions. Frege and Russell argued against that theory along the following lines: The transformation rules of the mathematical language do not determine the meaning of the mathematical signs. The meaning of these signs is only determined if they are explicitly defined with the help of the basic concepts of logic. According to *Logical Syntax*, Frege and Russell were right in claiming that the transformation rules of the mathematical language do not provide an interpretation of that language. However, whether an interpretation of the mathematical language into a total language is established by defining the mathematical signs or in some other way is a question devoid of philosophical significance. Thus, according to *Logical Syntax,* part of Frege and Russell's discussion of the theory of implicit definitions concerned a purely technical question.

regarded as central to classical logicism, and the formalist rejects it because of his method of distinguishing between mathematics and metamathematics.[25]

There is, however, only one plausible way to clarify this claim. It is suggested by the following remark made by Carnap:

> … we intend to show that, actually, it is possible to manage with one language only; not, however, by renouncing syntax, but by demonstrating that … the syntax of this language can be formulated within this language itself. (53)

But can we equate the idea of a universal language with the proposal that we should, in pursuing certain ends, use a language containing its own syntax? Obviously, such an attempt presupposes that it is possible to formulate the whole syntax of a language within that language. And this is, contrary to the impression conveyed by the remark just quoted, not the case for a consistent language. It is impossible, for example, to define "analytic in L" within a consistent language L (219). Considerations similar to those which establish that "analytic" is not definable in a consistent language even show directly, in Carnap's opinion, that the single language approach to mathematics is inacceptable. He writes in *Logical Syntax*:

> … there exists neither a language in which all arithmetical terms can be defined nor one in which all arithmetical sentences are resoluble. … In other words, *everything mathematical can be formalized, but mathematics cannot be exhausted by one system*; it requires an infinite series of ever richer languages. (222)

The logicist idea of an all-encompassing framework of discourse is therefore simply wrong, and, for this reason, this seemingly important aspect of classical logicism cannot be used to define a philosophically significant position.

8. Conclusion

In the previous chapters, I have discussed those aspects of Carnap's logicism in 1930 and of Frege-Russell logicism which might plausibly be regarded as being translatable

[25] Carnap accepted this idea until around 1930–31. See Reck (2004, 173).

into philosophically significant theses within the framework of *Logical Syntax*. If my argumentation is conclusive, we must judge the relation of Carnap's standpoint in 1934 to these logicist positions as follows:

First, essential aspects of Carnap's logicism in 1930 are incorporated into the very framework of general syntax: In combining Wittgenstein's analysis of the nature of logic with the reducibility of mathematics to logic, he tried to establish the thesis in 1930 that classical logic and mathematics are tautological and free of any factual content. In *Logical Syntax* this thesis leads, on the one hand, to the following requirement: The transformation rules of a calculus which is to contain classical mathematics must be chosen in such a way that, in accordance with them, all those sentences result from the null class which are regarded as correct in classical mathematics. On the other hand, that thesis leads to the requirement that the logico-mathematical sentences of any language be logically determined in that language, that is, that they be either analytic or contradictory. However, the reducibility of mathematics to logic no longer matters in 1934. Whereas this reducibility was necessary in 1930 in order to guarantee the tautological nature of mathematics, it is irrelevant to the project of *Logical Syntax* as a whole due to Carnap's general definition of analyticity.

Second, if we try to define a position within the framework of *Logical Syntax* by taking claims maintained in the logicist tradition as starting points, then the following proposal ensues:

(P) For certain purposes it is advisable to use a total language containing not only classical logic and mathematics, but also application rules for the use of logico-mathematical signs in synthetic sentences.

The proposal (P) is a formal rendering of the demand that mathematics be applicable to empirical matter as well as of the demand that the mathematical signs be given a definite, determinable meaning. Moreover, (P) also captures what remains of the core idea of classical logicism within the framework of *Logical Syntax*: the idea that classical mathematics is tautological because of its reducibility to pure logic. A position characterised with the help of (P) is therefore not only philosophically significant; it also captures aspects of Carnap's logicism in 1930 as well as of Frege-Russell logicism. In addition, the ideal of a universal language, the demand that the meaning of mathematical signs be determined through definition, and the idea that mathematics is part of logic lose their significance completely within the framework of *Logical Syntax*. Therefore, we must conclude that (P) is the best we can make of the logicist positions within that framework.[26]

However, one question still remains: Should we, as Carnap does, really regard the proposal (P) as typically logicist? The insistence on the need for an integral framework containing the formal as well as the empirical sciences is an important constant in Carnap's thought (cf. Awodey and Carus 2001, 153–154) and, on the one hand, it is clear that such an integrality requirement was emphasized, for example, by Frege.[27] Here we have therefore a continuity with one of the key figures in the logicist tradition, a fact Carnap was aware of. He writes in the chapter of his autobiography devoted to his first philosophical steps:

> … the following conception, which derives essentially from Frege, seemed to me of paramount importance: It is the task of logic and of mathematics within the total system of knowledge to supply the forms of concepts, statements, and inferences, forms which are then applicable everywhere … It follows from these considerations that the nature of logic and mathematics can be clearly understood only if close attention is given to their application in non-logical fields, especially in empirical science. (Carnap 1963, 12)

On the other hand, it is equally clear that the logicists were not the only ones who emphasised this integrality requirement. In particular, some of the neo-Kantians subscribed to such a requirement for mathematics as well. Cassirer, for example, maintained in 1907 that the real problem of epistemology is the role played by mathematical principles in our construction of reality. Philosophy therefore should not

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[26] Oberdan concludes in his paper *The Synthesis of Logicism and Formalism* that there remains little of interest in logicism within the framework of *Logical Syntax*. He justifies his conclusion on the basis of the following two claims: i) In 1934 Carnap rejects the logicist ideal of a universal language form. ii) According to *Logical Syntax,* it is a purely technical question whether the meaning of the mathematical signs is determined through explicit definitions or not. See Oberdan (1993, 162–167). These claims are both correct, and thus it is equally correct that important aspects of the original logicist concerns lose their importance in *Logical Syntax*. Nevertheless, Oberdan's verdict is too drastic. As has been shown in this paper, it is possible to define a philosophically significant position within the framework of *Logical Syntax* starting from claims championed by proponents of a logicist position.

[27] In *Grundgesetze*, Frege claims for example: "It is applicability alone which raises arithmetics above the game to the status of a science. Thus, applicability is an essential part." (Frege 1903, 100, my translation)

focus on either mathematics or physics alone, but rather on the connection between these two domains (Cassirer 1907, 48).[28]

There is therefore little reason to describe Carnap's integrality requirement as "typically logicist". Since all that remains of classical logicist concerns within the framework of *Logical Syntax* is the project to construe an integral language, we must thus conclude that there is simply no place for a philosophically significant position within that framework which really deserves the name "logicism".

Moreover, it is not only impossible to define a logicist position by such an integrality requirement, it is also questionable whether it is necessary to interpret this requirement from a logicist point of view in the context of Carnap's philosophy: Two facts, namely that Carnap's early philosophy was influenced by the writings of neo-Kantians such as Cassirer and that Cassirer also emphasised an integrality requirement for mathematics, suggest that Carnap's insistence on such a requirement can also be understood from the neo-Kantian background of his early philosophy. Further evidence for this supposition is provided by Carnap's complete awareness of being in accordance with the Marburg school when he emphasises the need for taking the application of mathematics into account: In *Der Raum*, he refers to Cassirer's paper *Kant und die moderne Mathematik* (Carnap 1922, 81, note to p. 61) and maintains that the formal geometric structures studied in mathematics are constructed solely for their application in physics (Carnap 1922, 61).

However, Carnap not only recognized that his insistence on an integrality requirement for mathematics was in agreement with Cassirer, but, at the very beginning of his philosophical career in 1920, he even pursued a project within which this requirement is intimately connected with elements characteristic of the neo-Kantian approach. Already at that time, Carnap had set himself the task of construing a unified system of all knowledge (cf. Carus 2007, 92). Since the formal sciences – deductive logic and mathematics – clearly are to be accommodated in such a system and are to be related within it somehow to the empirical realm, Carnap's insistence on an integrality requirement for mathematics can be seen as resulting from his more comprehensive demand for a unification and systematization of the whole of human knowledge. In addition, Carnap's endeavour to construct such a unified system in 1920 not only shows

[28] I am indebted to the anonymous referee of this journal for calling my attention to the fact that Cassirer emphasised an integrality requirement for mathematics as well.

that he has strong affinities with neo-Kantianism, but also that his conception of what constitutes such a system in the first place is even in conflict with Frege's logicism.

At that time Carnap maintained that a conceptual system for science has to be founded on and unified by a system of categories – categories which are presupposed by deductive logic itself and are to be investigated within a transcendental logic (cf. Carus 2007, 103–104). This does not mean that Frege's logicism was irrelevant for Carnap's project in 1920. Carnap's attempt to construct a unified conceptual system was inspired by the logicist thesis of the unity of deductive logic and mathematics (cf. Carus 2007, 102–104). Nonetheless, the idea that a system of all scientific concepts cannot be systematized by deductive logic alone but must be unified by a system of categories is incompatible with Frege's philosophy and is central to the thinking of such neo-Kantians as Natorp and Cassirer.[29]

I do not want to suggest that Carnap adopted his integrality requirement from these neo-Kantians and not from Frege. It is quite plausible to suppose that it was Frege, the logicist, who inspired Carnap to emphasize the importance of taking the application of mathematics into account. Nevertheless, it is possible to strengthen the conclusion established some paragraphs ago: Since an integrality requirement for mathematics is a natural consequence of Carnap's attempt to construct a unified and comprehensive system of knowledge within a system of categories, a clearly neo-Kantian project, we have not only to reject the claim that an integrality requirement for mathematics is typically logicist. We must even reject the claim that Carnap's insistence on this requirement clearly indicates the presence of an idea in his philosophical approach which is to be understood on the basis of his commitment to the logicist tradition.

[29] Natorp writes for example: "In particular, the unity of the sciences is to be based on the unity of their logical fundament, that is, not merely on the use of the same methods of thinking, but on the fact that the basic plan according to which their common object constitutes itself is presaged in the basic laws of object knowledge as such, which are to be exposed by logic." (Natorp 1910, 9, my translation) The same point is made by Cassirer, see for example Cassirer (1907, 45).

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